Name:__________________________

SM3 7.3 Log Graphs

The core of a logarithmic function is having a variable as the argument of a log.

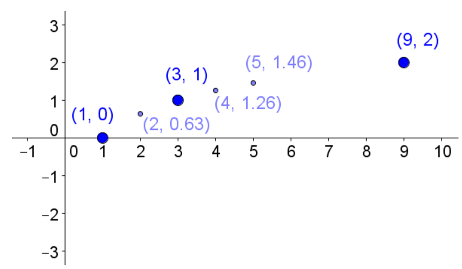
$f(x) = \log_3 x$ x is the variable as an argument

3 is the base of the log

We can produce a table of values to examine the behavior of the logarithmic function, just like we did with other functions.

x	$f(x)$	
$\vdots$ -2 -1 0	$\emptyset$	There is no power we can raise 3 to in order to produce a negative number or 0. Therefore, negative numbers and 0 are not a part of the domain.
***	***	(we'll discuss this below)
1	0	because $\log_3 1 = 0$
2	$\log_3 2$	because $\log_3 2$ doesn't simplify as 2 is not a power of 3
3	1	because $\log_3 3 = 1$
4	$\log_3 4$	because $\log_3 4$ doesn't simplify as 4 is not a power of 3
5	$\log_3 5$	because $\log_3 5$ doesn't simplify as 5 is not a power of 3
$\vdots$	$\vdots$	$\log_3 6, \log_3 7, \log_3 8$ won't simplify
9	2	because $\log_3 9 = 2$
$\vdots$	$\vdots$	
	3	For which x-value will $\log_3 x = 3$?

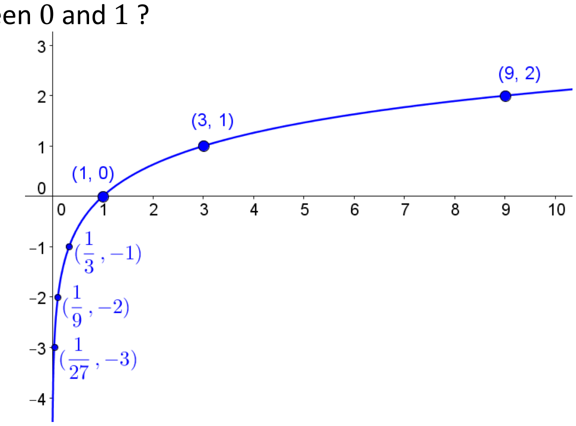
We plotted the simple points (large in the picture) and the points that required a calculator to be accurate (small in the picture).



Our table shows that any x-value that is negative or 0 can't be included on the domain of $f(x)$. What about the space between 0 and 1 ?

*** What about the powers of 3 that are between 0 and 1 ?

Which x-value allows $\log_3 x = -1$? Since $3^{-1} = \frac{1}{3}$, $\log_3\left(\frac{1}{3}\right) = -1$. That means we could have included the point $\left(\frac{1}{3}, -1\right)$ on our table. Likewise, we could have included $\left(\frac{1}{9}, -2\right), \left(\frac{1}{27}, -3\right), \cdots$ This means that the closer we get to $x = 0$, the more negative $f(x)$ becomes.



I added the first few of these points (and also got rid of the smaller points from above) in the figure to the right.

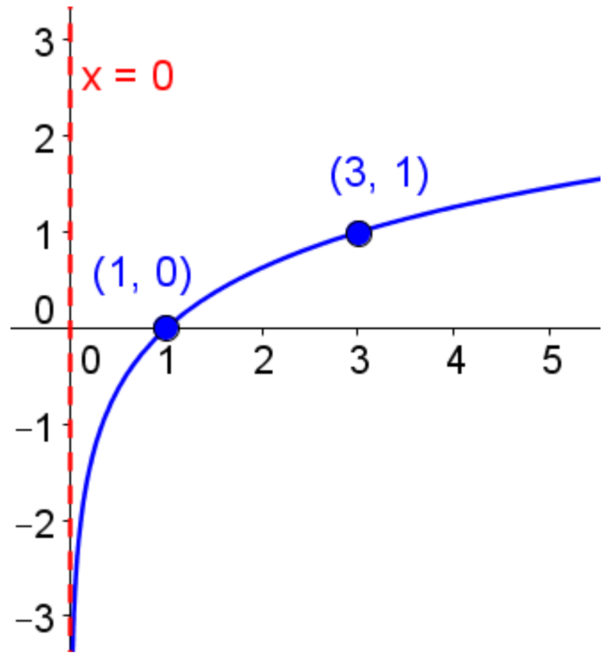
So, what the minimum we can do to represent the behavior of a log function graphically? If we include the points that required a calculator to be accurate, we've got 9 points to plot, which seems excessive.

Step 1) Sketch the vertical border of the log function. It'll be at $x = 0$ unless the function has been shifted horizontally. Whichever x-value simplifies to $\log_b(0)$ is the x-value you want. Use a vertical dashed line for the border.

Step 2) Plot the point with the x-value that simplifies to $\log_b(1)$. This will be one unit to the right of the border if the graph is not transformed.

Step 3) Plot the point with the x-value that simplifies to $\log_b(b)$. This will be b units to the right of the border if the graph is not transformed.

Step 4) Connect the points with a curve that runs close to the border.



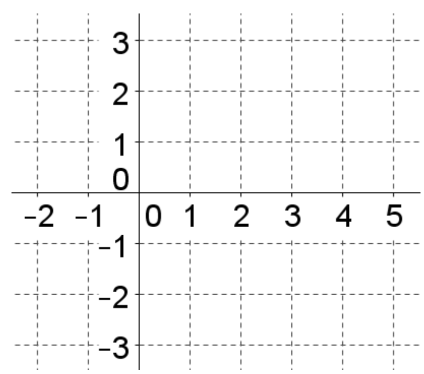
Example: Graph $f(x) = \log_4(x)$

Step 1) Sketch the vertical border with a dashed line at the x-value that causes $f(x) = \log_4(0)$.

Step 2) Plot the point with the x-value that causes $f(x) = \log_4(1)$.

Step 3) Plot the point with the x-value that causes $f(x) = \log_4(4)$.

Step 4) Sketch the curve.



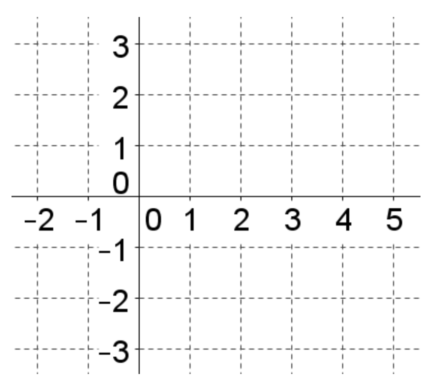
Example: Graph $f(x) = \log_2(x - 1)$

Step 1) Sketch the vertical border with a dashed line at the x-value that causes $f(x) = \log_2(0)$.

Step 2) Plot the point with the x-value that causes $f(x) = \log_2(1)$.

Step 3) Plot the point with the x-value that causes $f(x) = \log_2(2)$.

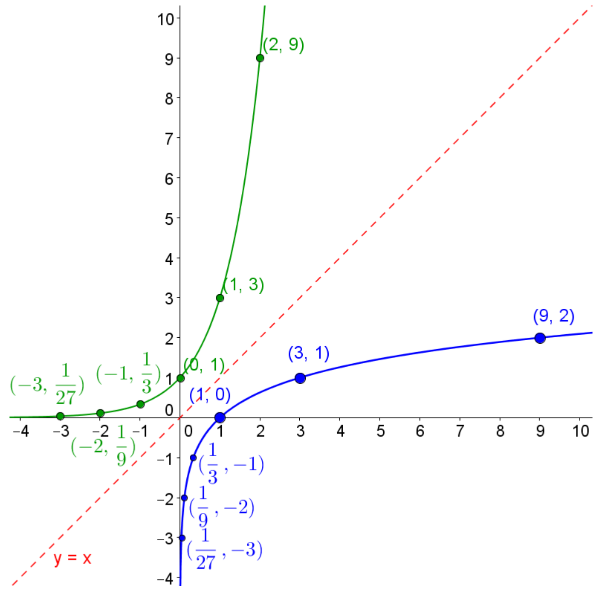
Step 4) Sketch the curve.



The logarithm operator is the inverse of the exponential operator. We can't be too surprised that the logarithm graph is the graphical inverse of the exponential graph.

$y = \log_3 x$ is a reflection of $y = 3^x$ over the line $y = x$ because the functions are inverses of one another.

Each point (a, b) on $y = 3^x$ has a reflection on $y = \log_3 x$ at (b, a).



Being the inverse of the exponential graph will make the log graph's properties easy to discern.

Property	Exponential $y = 3^x$	Logarithmic $f(x) = \log_3 x$
Domain	$\mathbb{R}$	$(0, \infty)$
Range	$(0, \infty)$	$\mathbb{R}$
	The original's domain is the inverse's range and the original's range is the inverse's domain.	
Interval of inc/dec	Increases on entire domain	Increases on entire domain
	When the original is increasing, the inverse is also increasing. Since the original is never decreasing, the inverse is also never decreasing.	
x-intercepts	$\emptyset$	$(0,1)$
y-intercepts	$(0,1)$	$\emptyset$
	The original's x-intercepts are the inverse's y-intercepts and the original's y-intercepts are the inverse's x-intercepts.	
Right End behavior	$\lim_{x\to\infty} y = \infty$	$\lim_{x\to\infty} y = \infty$
	When the original's x-values approach ∞, its y-values approach ∞; When the inverse's x-values approach ∞, its y-values approach ∞.	
Left End behavior	$\lim_{x\to-\infty} y = 0$	$\lim_{x\to 0} y = -\infty$ (*not end behavior)
	When the original's x-values approach $-\infty$, its y-values approach 0; When the inverse's x-values approach 0, its y-values approach $-\infty$. *$x = 0$ isn't the end of the axis; we omit this from end behavior discussion. So, logs will only have one end behavior limit discussion.	

Graph the following. Include the border and two points on the function.

1. $y = \log_4 x$

2. $y = -\log_2(x)$

3. $y = 2\log_4(x+3)$

4. $y = -2\log_4(x) + 1$

5. $y = \log_2(3 - x)$

6. $y = \frac{1}{2}\log_3(x)$

7. $y = 2\ln(x)$

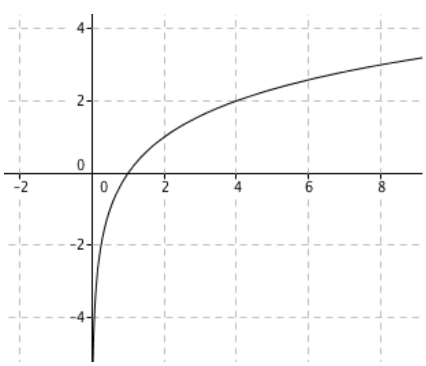
8. $y = \ln(x+1)$

9. $y = 2 + \ln(x)$

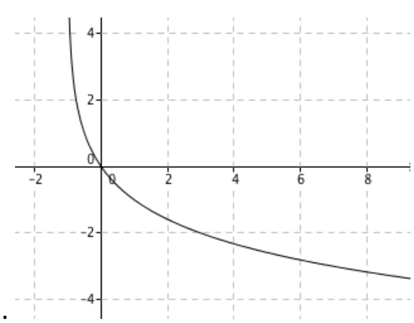
For the problems 7-16 find the listed properties. You may graph the function if you find that it helps you to see the properties:

a. domain
b. range
c. intervals of increase and decrease
d. zeroes
e. y-intercept
f. end behavior

10.



11.



12. $y = \log_2 x$

13. $y = -\log_3 x$

14. $y = 2\log x$

15. $y = 3ln(x) + 2$

16. $y = -ln(x - 2)$

17. $y = \ln(x - 1)$

18. $y = a\log_b x$, a and b are natural numbers greater than 1

19. $y = -a \log_b x$, a and b are natural numbers greater than 1

Find the average rate of change on the given interval.

20. $y = \log_2 x$ on $[4,8]$

21. $y = -\log_3 x$ on $\left[\frac{1}{3}, 3\right]$

22. $y = ln(x)^2$ on $[2,6]$